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ON NEUTROSOPHIC ϒ- OPEN SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract: We present a novel concept of neutrosophic sets namely neutrosophic ϒ-open and neutrosophic ϒ-closed sets in neutrosophic topological spaces. We also study in detail the properties of neutrosophic ϒ-open sets and its relation with other neutrosophic sets. In addition, we define and examine the attributes of neutrosophic ϒ-interior and neutrosophic ϒ-closure operators.

Keywords and Phrases: Neutrosophic ϒ-open, neutrosophic ϒ-closed, neutrosophic ϒ-interior, neutrosophic ϒ-closure.

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1. Introduction

Uncertainty is an essential and extensive factor in the real world which is too complicated to handle. Numerous theories evolved as mathematical approaches to deal with uncertainties. Zadeh [25] proposed the concept of fuzzy set in 1965 with the idea that each element has a degree of membership. Later, Atanassov [3] in 1986 introduced intuitionistic fuzzy sets as a generalization of fuzzy sets including the degree of non-membership with a restriction that the sum of these two grades is less than or equal to unity. Smarandache [21] in 1998 initiated the concepts of neutrosophic sets which is characterized by a truth membership function, an indeterminacy membership function and falsity membership function. This theory is highly significant in many application domains since indeterminacy is ubiquitous and these membership functions are important. In 2012, this significant concept was wielded in topology and conceptualized as neutrosophic topological spaces by Salama et.al [18]. Consequently, Karatas and Kuru [12] defined and characterized some basic topological notions such as interior, closure and subspaces. Ray and Sudeep [16] proposed the definitions of neutrosophic point and neighbourhood structure. In 2016, Iswarya and Bageerathi [11] introduced the concept of neutrosophic semi-open and semi-closed sets in neutrosophic topological spaces. Later the notion of generalized closed sets in neutrosophic topological spaces were introduced by Dhavaseelan and Saied Jafari [5]. Many other structures have also been defined on neutrosophic sets using the closure and interior operator and their topological properties are being studied. In addition, numerous applications of neutrosophic theory in medical diagonosis [4], decision making problems [14], image processing [2, 8, 20], safety modelling [23] etc., were also explored in the recent years. The focus of this paper is to introduce a new class of neutrosophic sets namely neutrosophic Υ-open (Υ-upsilon) sets and neutrosophic Υ-closed sets in neutrosophic topological space. We also observe its characterizations and relation with other neutrosophic sets. In addition, we define neutrosophic Υ-interior and neutrosophic Υ-closure and study some of its properties.

2. Preliminaries

In this section, we have presented some basic notions and results required for the progression of this work. Throughout this work,N_{tr}set refers to neutrosophic set.

Definition 2.1. [18] *Let U be a non-empty fixed set. A neutrosophic set L is an object having the form $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ where $\mu_L(u), \sigma_L(u)$ and $\gamma_L(u)$ represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element $u \in U$. A neutrosophic set $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ can be identified to an ordered triple $< \mu_L, \sigma_L, \gamma_L >$ in $]^-0, 1^+[$ on U.*

Definition 2.2. [18] *Let U be a non-empty set and $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}, M = \{< u, \mu_M(u), \sigma_M(u), \gamma_M(u) >: u \in U\}$ be neutrosophic sets in U. Then*

i. $L \subseteq M$ *if* $\mu_L(u) \leq \mu_M(u), \sigma_L(u) \leq \sigma_M(u)$ *and* $\gamma_L(u) \geq \gamma_M(u)$ *for all* $u \in U$.

ii. $L \cup M = \{< u, max\{\mu_L(u), \mu_M(u)\}, max\{\sigma_L(u), \sigma_M(u)\}, min\{\gamma_L(u), \gamma_M(u)\} >: u \in U\}$

iii. $L \cap M = \{< u, min\{\mu_L(u), \mu_M(u)\}, min\{\sigma_L(u), \sigma_M(u)\}, max\{\gamma_L(u), \gamma_M(u)\} >:$

$u \in U\}$

iv. $L^c = \{< u, \gamma_L(u), 1 - \sigma_L(u), \mu_L(u) >: u \in U\}$

v. $0_{N_{tr}} = \{< u, 0, 0, 1 >: u \in U\}$ *is the neutrosophic empty set and* $1_{N_{tr}} = \{< u, 1, 1, 0 >: u \in U\}$ *is the neutrosophic whole set.*

Definition 2.3. [18] *A* ***neutrosophic topology*** *on a non-empty set* U *is a family* $\tau_{N_{tr}}$ *of neutrosophic sets in* U *satisfying the following axioms:*

i. $0_{N_{tr}}, 1_{N_{tr}} \in \tau_{N_{tr}}$.

ii. $\cup L_i \in \tau_{N_{tr}} \forall \{L_i : i \in I\} \subseteq \tau_{N_{tr}}$

iii. $L_1 \cap L_2 \in \tau_{N_{tr}}$ *for any* $L_1, L_2 \in \tau_{N_{tr}}$

The pair $(U, \tau_{N_{tr}})$ *is called a neutrosophic topological space. The members of* $\tau_{N_{tr}}$ *are called neutrosophic open*$(N_{tr}O)$ *and its complements are called neutrosophic closed* $(N_{tr}C)$.

Definition 2.4. [16] *A neutrosophic set* $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ *is called a* ***neutrosophic point*** *if for any element* $v \in U, \mu_L(v) = a, \sigma_L(v) = b, \gamma_L(v) = c$ *for* $u = v$ *and* $\mu_L(v) = 0, \sigma_L(v) = 0, \gamma_L(v) = 1$ *for* $u \neq v$, *where* a, b, c *are real standard or non standard subsets of* $]^-0, 1^+[$. *A neutrosophic point is denoted by* $u_{a,b,c}$. *For the neutrosophic point* $u_{a,b,c}, u$ *will be called its support.*

Definition 2.5. [18] *Let* $(U, \tau_{N_{tr}})$ *be a neutrosophic topological space and* $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ *be a neutrosophic set in* U. *Then the neutrosophic closure and neutrosophic interior of* L *are defined as*

i. $N_{tr}cl(L) = \bigcap\{F: F$ *is neutrosophic closed set in* U *and* $L \subseteq F\}$

ii. $N_{tr}int(L) = \bigcup\{G: G$ *is neutrosophic open set in* U *and* $G \subseteq L\}$

Definition 2.6. *A neutrosophic topological space* $(U, \tau_{N_{tr}})$ *is said to be* ***neutrosophic locally indiscrete*** *if every neutrosophic open set is neutrosophic closed.*

Definition 2.7. *A neutrosophic set* L *of a neutrosophic topological space* $(U, \tau_{N_{tr}})$ *is*

i. ***neutrosophic semi-open*** [11] *if* $L \subseteq N_{tr}cl(N_{tr}int(L))$ *and* ***neutrosophic semi-closed*** *if* $N_{tr}int(N_{tr}cl(L)) \subseteq L$.

ii. ***neutrosophic pre-open*** [24] *if* $L \subseteq N_{tr}int(N_{tr}cl(L))$ *and* ***neutrosophic pre-closed*** *if* $N_{tr}cl(N_{tr}int(L)) \subseteq L$.

iii. ***neutrosophic α-open*** [1] *if $L \subseteq N_{tr}int(N_{tr}cl(N_{tr}int(L)))$ and* ***neutrosophic α-closed*** *if $N_{tr}cl(N_{tr}int(N_{tr}cl(L))) \subseteq L$.*

iv. ***neutrosophic semi-preopen*** *or β-open* [17] *if $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$ and* ***neutrosophic semi-preclosed*** *or β-closed if $N_{tr}int(N_{tr}cl(N_{tr}int(L))) \subseteq L$.*

v. ***neutrosophic b-open*** [7] *if $L \subseteq N_{tr}cl(N_{tr}int(L)) \cup N_{tr}int(N_{tr}cl(L))$ and* ***neutrosophic b-closed*** *if $N_{tr}cl(N_{tr}int(L)) \cap N_{tr}int(N_{tr}cl(L)) \subseteq L$.*

The class of all neutrosophic semi-open (respectively neutrosophic pre-open, neutrosophic α-open, neutrosophic β-open, neutrosophic b-open) sets are denoted by $N_{tr}SO(U, \tau_{N_{tr}})$ (respectively $N_{tr}PO(U, \tau_{N_{tr}}), N_{tr}\alpha O(U, \tau_{N_{tr}}), N_{tr}\beta O(U, \tau_{N_{tr}})$, $N_{tr}bO(U, \tau_{N_{tr}})$).

Definition 2.8. *A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be*

i. ***neutrosophic generalized closed***$(N_{tr}gC)$ [5] *if $N_{tr}cl(L) \subseteq G$ whenever $L \subseteq G$ and G is neutrosophic open in U.*

ii. ***neutrosophic generalized semi-closed***$(N_{tr}gsC)$ [9] *if $N_{tr}scl(L) \subseteq G$ whenever $L \subseteq G$ and G is neutrosophic open in U.*

iii. ***neutrosophic generalized b-closed***$(N_{tr}gbC)$ [13] *if $N_{tr}bcl(L) \subseteq G$ whenever $L \subseteq G$ and G is neutrosophic open in U.*

iv. ***neutrosophic ψ-closed***$(N_{tr}\psi C)$ [15] *if $N_{tr}scl(L) \subseteq G$ whenever $L \subseteq G$ and G is neutrosophic sg-open in U.*

Theorem 2.9. [13] *A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is neutrosophic generalized b-open if and only if $F \subseteq N_{tr}bint(L)$ whenever $F \subseteq L$ and F is neutrosophic closed.*

Theorem 2.10. [9] *A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is neutrosophic generalized semi-open if and only if $F \subseteq N_{tr}sint(L)$ whenever $F \subseteq L$ and F is neutrosophic closed.*

Theorem 2.11. [10] *Let L be a neutrosophic set in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then the following conditions hold:*

i. $N_{tr}scl(L) = L \cup N_{tr}int(N_{tr}cl(L))$

ii. $N_{tr}sint(L) = L \cap N_{tr}cl(N_{tr}int(L))$

Theorem 2.12. [7] *Let L be a neutrosophic set in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then*

i. $N_{tr}bcl(L) = N_{tr}scl(L) \cap N_{tr}pcl(L)$

ii. $N_{tr}bint(L) = N_{tr}sint(L) \cup N_{tr}pint(L)$

3. Neutrosophic Υ-open Sets

This section conceptualizes the idea of neutrosophic Υ-open sets in neutrosophic topological spaces. Moreover, a detailed characterization of this new class of sets has been presented through results and illustrations.

Definition 3.1. *A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be **neutrosophic Υ-open** if for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$. The class of neutrosophic Υ-open sets is denoted by $N_{tr}\Upsilon O(U, \tau_{N_{tr}})$.*

Example 3.2. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L, M\}$ where $L = \{< a, 0.5, 0.6, 0.1 >< b, 0.6, 0.7, 0.2 >\}$ and $M = \{< a, 0.8, 0.6, 0 >< b, 0.7, 0.8, 0.1 >\}$. Consider the collection $\mathcal{P} = \{P \colon M \subset P \subset 1_{N_{tr}}\}$ of neutrosophic sets in U. Then, $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, M, \mathcal{P}, 1_{N_{tr}}\}$.

Remark 3.3. *The class of neutrosophic Υ-open sets is undefined in indiscrete topological spaces. Henceforth, we consider the neutrosophic topological spaces excluding the indiscrete topological space.*

Remark 3.4. *In any neutrosophic topological space $(U, \tau_{N_{tr}}), 0_{N_{tr}}$ and $1_{N_{tr}}$ are $N_{tr}\Upsilon$-open.*

Theorem 3.5. *The union of an arbitrary collection of $N_{tr}\Upsilon$-open sets is also $N_{tr}\Upsilon$-open.*

Proof. Let $\{L_i : i \in I\}$ be a collection of $N_{tr}\Upsilon$-open sets. Then for each $i \in I$,L_i is a $N_{tr}\Upsilon$-open set which implies $L_i \subseteq N_{tr}cl(N_{tr}int(L_i \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Now, $\cup_{i\in I} L_i \subseteq \cup_{i\in I} N_{tr}cl(N_{tr}int(L_i \cup F)) \subseteq N_{tr}cl(\cup_{i\in I} N_{tr}int(L_i \cup F)) \subseteq N_{tr}cl(N_{tr}int(\cup_{i\in I}(L_i \cup F)))$. Therefore, $\cup_{i\in I} L_i \subseteq N_{tr}cl(N_{tr}int(\cup_{i\in I} L_i \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Hence $\cup_{i\in I} L_i$ is a $N_{tr}\Upsilon$-open set.

Remark 3.6. *The intersection of two $N_{tr}\Upsilon$-open sets need not be $N_{tr}\Upsilon$-open.*

Example 3.7. Let $U = \{a, b\}$ and
$L_1 = \{< a, 0.1, 0.3, 0.8 >< b, 0.2, 0.5, 0.7 >\}$
$L_2 = \{< a, 0.1, 0.4, 0.8 >< b, 0.9, 0.5, 0.7 >\}$
$L_3 = \{< a, 0.7, 0.4, 0.4 >< b, 0.9, 1, 0.3 >\}$

$L_4 = \{< a, 0.7, 0.3, 0.4 >< b, 0.2, 1, 0.3 >\}$
$L_5 = \{< a, 0.2, 0.5, 0.7 >< b, 0.9, 0.6, 0.3 >\}$
$L_6 = \{< a, 0.9, 0.7, 0.4 >< b, 0.4, 1, 0.2 >\}$
$L_7 = \{< a, 0.2, 0.5, 0.7 >< b, 0.4, 0.6, 0.3 >\}$ be neutrosophic sets in U.
Then $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L_1, L_2, L_3, L_4\}$ forms a neutrosophic topology on U. Here, L_5 and L_6 are $N_{tr}\Upsilon$-open but their intersection L_7 is not $N_{tr}\Upsilon$-open.

Remark 3.8. *The class of $N_{tr}\Upsilon$-open sets does not form a topology on U.*

Theorem 3.9. *If L is a $N_{tr}\Upsilon$-open set of a neutrosophic topological space $(U, \tau_{N_{tr}})$, then for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$, there exists a N_{tr}open set G such that $G \subseteq L \cup F \subseteq N_{tr}cl(G \cup F)$.*
Proof. Suppose L is $N_{tr}\Upsilon$-open. Then $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non empty N_{tr}closed set $F \neq 1_{N_{tr}}$. It is obvious that $N_{tr}int(L \cup F) \subseteq L \cup F$. Now $L \cup F \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \cup F \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \cup N_{tr}cl(F) = N_{tr}cl(N_{tr}int(L \cup F) \cup F)$. Hence, for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$, there exists a N_{tr}open set $G = N_{tr}int(L \cup F)$ such that $G \subseteq L \cup F \subseteq N_{tr}cl(G \cup F)$.

Remark 3.10. *The converse of the above theorem need not be true.*

Example 3.11. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.8, 0.6, 0 >< b, 0.7, 0.8, 0.1 >\}$. The N_{tr}closed sets are $0_{N_{tr}}, 1_{N_{tr}}$ and L^c. Consider the collections $\mathcal{P} = \{P\colon L \subset P \subset 1_{N_{tr}}\}$ and $\mathcal{Q} = \{P^c\colon P \in \mathcal{P}\}$ of neutrosophic sets in U. Here, $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, 1_{N_{tr}}\}$. Now, there exists a N_{tr} open set $0_{N_{tr}}$ such that $0_{N_{tr}} \subseteq P^c \cup L^c \subseteq N_{tr}cl(0_{N_{tr}} \cup L^c)$ but P^c is not $N_{tr}\Upsilon$-open.

The converse holds if $(U, \tau_{N_{tr}})$ is a neutrosophic locally indiscrete space as shown in the following theorem.

Theorem 3.12. *Let $(U, \tau_{N_{tr}})$ be a neutrosophic locally indiscrete space. Then L is $N_{tr}\Upsilon$-open in $(U, \tau_{N_{tr}})$ if and only if for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$, there exists a N_{tr}open set G such that $G \subseteq L \cup F \subseteq N_{tr}cl(G \cup F)$.*
Proof. Necessity follows from theorem 3.9. Now, we shall prove the sufficient condition. Suppose for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$, there exists a N_{tr}open set G such that $G \subseteq L \cup F \subseteq N_{tr}cl(G \cup F)$. Now, $G \subseteq L \cup F$ implies $G = N_{tr}int(G) \subseteq N_{tr}int(L \cup F)$. Also, since $(U, \tau_{N_{tr}})$ is a neutrosophic locally indiscrete space, F is N_{tr}open. Hence, $L \subseteq L \cup F \subseteq N_{tr}cl(G \cup F) \subseteq N_{tr}cl(N_{tr}int(L \cup F) \cup N_{tr}int(F)) \subseteq N_{tr}cl(N_{tr}int(L \cup F))$.Therefore $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non empty N_{tr}closed set $F \neq 1_{N_{tr}}$ which implies L is $N_{tr}\Upsilon$-open.

Theorem 3.13. *A neutrosophic set L in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon$-open if and only if for every neutrosophic point $u_{a,b,c} \in L$, there exists a*

$N_{tr}\Upsilon$-open set $M_{u_{a,b,c}}$ such that $u_{a,b,c} \in M_{u_{a,b,c}} \subseteq L$.
Proof. If L is $N_{tr}\Upsilon$-open , then we may consider $M_{u_{a,b,c}} = L$ for every $u_{a,b,c} \in L$. Conversely, assume that for every $u_{a,b,c} \in L$, there exists a $N_{tr}\Upsilon$-open set $M_{u_{a,b,c}}$ such that $u_{a,b,c} \in M_{u_{a,b,c}} \subseteq L$. Then, $L = \cup\{u_{a,b,c} \colon u_{a,b,c} \in L\} \subseteq \cup\{M_{u_{a,b,c}} \colon u_{a,b,c} \in L\} \subseteq L$. Hence, by theorem 3.5, $L = \cup\{M_{u_{a,b,c}} \colon u_{a,b,c} \in L\}$ is $N_{tr}\Upsilon$-open.

Theorem 3.14. *Every N_{tr}open set is $N_{tr}\Upsilon$-open.*
Proof. Let L be a N_{tr}open set in U. Then, we have $L = N_{tr}int(L) \subseteq N_{tr}int(L \cup F)$ which implies $N_{tr}cl(L) \subseteq N_{tr}cl(N_{tr}int(L \cup F))$. Therefore $L \subseteq N_{tr}cl \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$ which implies L is $N_{tr}\Upsilon$-open.

Remark 3.15. *The converse of the above theorem need not be true.*

Example 3.16. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.7, 0.6, 0.8 >< b, 0.6, 0.8, 0.9 >\}$.Consider the collections $\mathcal{P} = \{P \colon L \subset P; L^c \subset P\}$and $\mathcal{Q} = \{Q \colon L \subset Q; Q \not\subset L^c; L^c \not\subset Q\}$ of neutrosophic sets in U.Then, $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}$ and $\mathcal{Q}$ are $N_{tr}\Upsilon$-open but not N_{tr}open.

Theorem 3.17. *Every N_{tr}semi-open set is $N_{tr}\Upsilon$-open.*
Proof. Let L be a N_{tr}semi-open set in U. Then,$L \subseteq N_{tr}cl(N_{tr}int(L)) \subseteq N_{tr}cl(N_{tr} int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$ which implies $L$is $N_{tr}\Upsilon$-open.

Remark 3.18. *The converse of the above theorem need not be true.*

Example 3.19. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L, M\}$ where $L = \{< a, 0.6, 0.2, 0.7 >< b, 0.5, 0.3, 0.7 >\}$ and $M = \{< a, 0.4, 0.1, 0.8 >< b, 0.4, 0.2, 0.9 >\}$. Consider the collections $\mathcal{P} = \{P \colon 0_{N_{tr}} \subset P \subset M\}, \mathcal{Q} = \{Q \colon L \not\subset Q; Q \not\subset L; Q \subset L^c\}$ and $\mathcal{R} = \{R \colon L \subset R \subset L^c\}$ of neutrosophic sets in U. Then $N_{tr}SO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, M, L^c, \mathcal{R}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, M, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}$ and $\mathcal{Q}$ are $N_{tr}\Upsilon$-open but not N_{tr}semi-open.

Theorem 3.20. *Every $N_{tr}\alpha$-open set is $N_{tr}\Upsilon$-open.*
Proof. Let L be a $N_{tr}\alpha$-open set in U. Then, we have $L \subseteq N_{tr}int(N_{tr}cl(N_{tr}int(L))) \subseteq N_{tr}cl(N_{tr}int(L)) \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$ which implies L is $N_{tr}\Upsilon$-open.

Remark 3.21. *The converse of the above theorem need not be true.*

Example 3.22. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.2, 0.3, 0.1 >< b, 0.3, 0.4, 0.9 >\}$. Consider the collections $\mathcal{P} = \{P \colon 0_{N_{tr}}$

$\subset P \subset L\}, \mathcal{Q} = \{Q\colon L \not\subset Q; Q \not\subset L; Q \subset L^c\}$ and $\mathcal{R} = \{R\colon L \subset R \subset L^c\}$ of neutrosophic sets in U. Then $N_{tr}\alpha O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$. Here, L^c and the neutrosophic sets in $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{R}$ are $N_{tr}\Upsilon$-open but not $N_{tr}\alpha$-open.

Theorem 3.23. *Every $N_{tr}\Upsilon$-open set is $N_{tr}\beta$-open.*

Proof. Let L be a non-empty $N_{tr}\Upsilon$-open set in U. We prove this result in two cases:

Case 1: $N_{tr}cl(L) \neq 1_{N_{tr}}$

Since L is $N_{tr}\Upsilon$-open, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Then there exists some $F = N_{tr}cl(L)$ such that $L \subseteq N_{tr}cl(N_{tr}int(L \cup N_{tr}cl(L)))$ which implies $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$.

Case 2: $N_{tr}cl(L) = 1_{N_{tr}}$

Since L is $N_{tr}\Upsilon$-open, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Then, $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L \cup F))) \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L) \cup N_{tr}cl(F)))$. Since $N_{tr}cl(L) = 1_{N_{tr}}$, $L \subseteq N_{tr}cl(N_{tr}int(1_{N_{tr}} \cup N_{tr}cl(F))) \subseteq N_{tr}cl(N_{tr}int (1_{N_{tr}})))$ and therefore $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$.

Hence $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$ in both the cases which implies L is $N_{tr}\beta$-open.

Remark 3.24. *The converse of the above theorem need not true.*

Example 3.25. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.7, 0.6, 0.2 >< b, 0.3, 0.8, 0.1 >\}$.Consider the collections $\mathcal{P} = \{P\colon L^c \subset P \subset L\}, \mathcal{Q} = \{Q\colon L \subset Q \subset 1_{N_{tr}}\}, \mathcal{R} = \{R\colon L^c \not\subset R; R \not\subset L^c; R \subset L\}, \mathcal{S} = \{S\colon L^c \not\subset S; S \not\subset L^c; S \not\subset L\}$and $\mathcal{T} = \{T\colon L^c \subset T \not\subset L\}$ of neutrosophic sets in U.Then, $N_{tr}\beta O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{Q}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}, \mathcal{R}, \mathcal{S}$ and $\mathcal{T}$ are $N_{tr}\beta$-open but not $N_{tr}\Upsilon$-open.

Theorem 3.26.

i. *Every $N_{tr}\Upsilon$-open set is $N_{tr}gs$-open.*

ii. *Every $N_{tr}\Upsilon$-open set is $N_{tr}gb$-open.*

Proof. (i) Let L be $N_{tr}\Upsilon$-open set in U. Then, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Suppose $F \subseteq L$. Then $F = L \cap F \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \cap F = N_{tr}cl(N_{tr}int(L)) \cap F \subseteq N_{tr}cl(N_{tr}int(L)) \cap L$. Now, by theorem 2.11, $F \subseteq N_{tr}cl(N_{tr}int(L)) \cap L = N_{tr}sint(L)$. Hence $F \subseteq N_{tr}sint(L)$ whenever $F \subseteq L$ and F is N_{tr}closed. Then by theorem 2.10, L is $N_{tr}gs$-open.

(ii) Let L be $N_{tr}\Upsilon$-open set in U. Then, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Suppose $F \subseteq L$.Then $F = L \cap F \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \cap F = N_{tr}cl(N_{tr}int(L)) \cap F \subseteq N_{tr}cl(N_{tr}int(L)) \cap L = N_{tr}sint(L)$.This implies

$F \subseteq N_{tr}sint(L) \cup N_{tr}pint(L)$ and by theorem 2.12, $F \subseteq N_{tr}bint(L)$ whenever $F \subseteq L$ and F is N_{tr}closed. Then, by theorem 2.9, L is $N_{tr}gb$-open.

Remark 3.27. *The converse of each of the statements in the above theorem need not be true.*

Example 3.28. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.5, 0.6, 0.1 >< b, 0.6, 0.7, 0.2 >\}$. Consider the collections $\mathcal{P} = \{P : 0_{N_{tr}} \subset P \subset L^c\}, \mathcal{Q} = \{Q : L^c \not\subset Q; Q \not\subset L^c; Q \subset L\}, \mathcal{R} = \{R : L^c \not\subset R; R \not\subset L^c; R \subset L\}$ and $\mathcal{S} = \{S : L \subset S \subset 1_{N_{tr}}\}$ of neutrosophic sets in U. Then $N_{tr}gsO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, 1_{N_{tr}}\}$and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{S}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}, \mathcal{Q}$ and $\mathcal{R}$ are $N_{tr}gs$-open but not $N_{tr}\Upsilon$-open.

Example 3.29. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.8, 0.9, 0.6 >< b, 0.7, 0.8, 0.9 >\}$. Consider the collections $\mathcal{P} = \{P : P \subset L, P \subset L^c\}, \mathcal{Q} = \{Q : L \subset Q; L^c \subset Q\}, \mathcal{R} = \{R : R \subset L; R \not\subset L^c; L^c \not\subset R\}, \mathcal{S} = \{S : L \not\subset S; S \not\subset L; L^c \subset S\}, \mathcal{T} = \{T : L \subset T; T \not\subset L^c; L^c \not\subset T\}, \mathcal{V} = \{V : V \not\subset L; L \not\subset V; V \subset L^c\}$and $\mathcal{W} = \{W : W \not\subset L; L \not\subset W; W \not\subset L^c; L^c \not\subset W\}$ of neutrosophic sets in U. Then $N_{tr}gbO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mathcal{V}, \mathcal{W}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{Q}, \mathcal{T}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}, \mathcal{R}, \mathcal{S}, \mathcal{V}$ and $\mathcal{W}$ are $N_{tr}gb$-open but not $N_{tr}\Upsilon$-open.

Theorem 3.30. *In any neutrosophic topological space* $(U, \tau_{N_{tr}}), N_{tr}SO(U, \tau_{N_{tr}}) \subseteq N_{tr}\Upsilon O(U, \tau_{N_{tr}}) \subseteq N_{tr}gsO(U, \tau_{N_{tr}})$.*That is the class of* $N_{tr}\Upsilon$*-open sets lie between* N_{tr}*semi-open sets and* N_{tr} *gs-open sets.*

Proof. Proof follows from theorem 3.17 and 3.26(i).

Remark 3.31. *The concepts of* $N_{tr}\Upsilon$*-open and* N_{tr}*pre-open are independent.*

Example 3.32. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.3, 0.4, 0.9 >< b, 0.4, 0.5, 0.8 >\}$. Consider the collections $\mathcal{P} = \{P : 0_{N_{tr}} \subset P \subset L\}, \mathcal{Q} = \{Q : L \subset Q \subset L^c\}, \mathcal{R} = \{R : L^c \subset R \subset 1_{N_{tr}}\}, \mathcal{S} = \{S : L \not\subset S; S \not\subset L;\ S \subset L^c\}$, $\mathcal{T} = \{T : L \not\subset T; T \not\subset L; T \not\subset L^c\}$ and $\mathcal{V} = \{V : L \subset V \not\subset L^c\}$ of neutrosophic sets in U. Then $N_{tr}PO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{R}, \mathcal{T}, \mathcal{V}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{S}, 1_{N_{tr}}\}$. Here, L^c and the neutrosophic sets in $\mathcal{Q}$ and $\mathcal{S}$ are $N_{tr}\Upsilon$-open but not N_{tr}pre-open. The neutrosophic sets in $\mathcal{R}, \mathcal{T}$ and $\mathcal{V}$ are N_{tr}pre-open but not $N_{tr}\Upsilon$-open.

Remark 3.33. *The concepts of* $N_{tr}\Upsilon$*-open and* $N_{tr}b$*-open are independent.*

Example 3.34. In example 3.32, $N_{tr}bO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{T}, \mathcal{V}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{S}$ are $N_{tr}\Upsilon$-open but not $N_{tr}b$-open and the neutrosophic sets in $\mathcal{R}, \mathcal{T}$ and $\mathcal{V}$ are $N_{tr}b$-open but not $N_{tr}\Upsilon$-open.

Remark 3.35. *The concepts of* $N_{tr}\Upsilon$*-open and* $N_{tr}g$*-open are independent.*

Example 3.36. Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.7, 0.8, 0.9 >< b, 0.7, 0.8, 0.9 >\}$. Consider the collections $\mathcal{P} = \{P \colon P \subset L, P \subset L^c\}, \mathcal{Q} = \{Q \colon Q \subset L; Q \not\subset L^c; L^c \not\subset Q\}, \mathcal{R} = \{R \colon L \subset R; R \not\subset L^c; L^c \not\subset R\}, \mathcal{S} = \{S \colon S \not\subset L; L \not\subset S; S \subset L^c\}, \mathcal{T} = \{T \colon T \not\subset L; L \not\subset T; T \not\subset L^c; L^c \not\subset T\}$ and $\mathcal{V} = \{V \colon L \subset V; L^c \subset V\}$ of neutrosophic sets in U. Then $N_{tr}gO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{R}, \mathcal{V}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{P}, \mathcal{Q}, \mathcal{S}$ and $\mathcal{T}$ are $N_{tr}g$-open but not $N_{tr}\Upsilon$-open and the neutrosophic sets in $\mathcal{V}$ are $N_{tr}\Upsilon$-open but not $N_{tr}g$-open.

Remark 3.37. *The concepts of $N_{tr}\Upsilon$-open and $N_{tr}\psi$-open are independent.*

Example 3.38. (i) Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.3, 0.2, 0.7 >< b, 0.4, 0.5, 0.6 >\}$. Consider the collections $\mathcal{P} = \{P \colon 0_{N_{tr}} \subset P \subset L\}, \mathcal{Q} = \{Q \colon L \not\subset Q; Q \not\subset L; Q \subset L^c\}$ and $\mathcal{R} = \{R \colon L \subset R \subset L^c\}$ of neutrosophic sets in U.Then $N_{tr}\psi O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{R}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$.Here, the neutrosophic sets in $\mathcal{Q}$ are $N_{tr}\Upsilon$-open but not $N_{tr}\psi$-open.
(ii) Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.5, 0.7, 0.6 >< b, 0.2, 0.4, 0.4 >\}$. Consider the collections $\mathcal{P} = \{P \colon L \subset P; L^c \subset P\}, \mathcal{Q} = \{Q \colon L \subset Q; Q \not\subset L^c; L^c \not\subset Q\}$ and $\mathcal{R} = \{R \colon R \subset L; R \not\subset L^c; L^c \not\subset R\}$ of neutrosophic sets in U. Then $N_{tr}\psi O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, 1_{N_{tr}}\}$. Here, the neutrosophic sets in $\mathcal{R}$ are $N_{tr}\psi$-open but not $N_{tr}\Upsilon$-open. From the above discussions, we have the following diagram

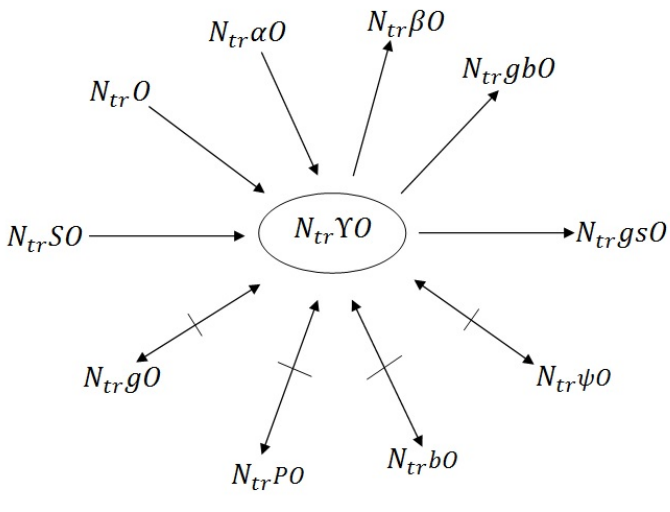

Figure 1

Theorem 3.39. *Let $(U, \tau_{N_{tr}})$ be a neutrosophic locally indiscrete space. Then, every $N_{tr}\Upsilon$-open set is N_{tr}pre-open and $N_{tr}b$-open.*
Proof. Let L be a $N_{tr}\Upsilon$-open set in U. Then, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Since $(U, \tau_{N_{tr}})$ is a neutrosophic locally indiscrete space, $L \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \subseteq N_{tr}cl(L \cup F) = N_{tr}int(N_{tr}cl(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Hence $L \subseteq N_{tr}int(N_{tr}cl(L \cup F))$ for every non-empty N_{tr}open set $F \neq 1_{N_{tr}}$.Then there exists some $F = N_{tr}int(L)$ such that $L \subseteq N_{tr}int(N_{tr}cl(L \cup N_{tr}int(L)))$ which implies $L \subseteq N_{tr}int(N_{tr}cl(L))$. Hence L is N_{tr}pre-open. Now, since $L \subseteq N_{tr}int(N_{tr}cl(L)), L \subseteq N_{tr}int(N_{tr}cl(L)) \cup N_{tr}cl(N_{tr}int(L))$. Hence L is $N_{tr}b$-open.

Theorem 3.40. *Let L be a $N_{tr}\Upsilon$-open set and M be a neutrosophic set in a neutrosophic topological space $(U, \tau_{N_{tr}})$ such that $L \subseteq M \subseteq N_{tr}cl(L)$. Then M is $N_{tr}\Upsilon$-open.*
Proof. Since $L \subseteq M, L \cup F \subseteq M \cup F$ and hence $N_{tr}cl(N_{tr}int(L \cup F)) \subseteq N_{tr}cl(N_{tr}int(M \cup F))$. Also, since $M \subseteq N_{tr}cl(L)$ and L is $N_{tr}\Upsilon$-open, $M \subseteq N_{tr}cl(N_{tr}cl(N_{tr}int(L \cup F))) \subseteq N_{tr}cl(N_{tr}int(L \cup F)) \subseteq N_{tr}cl(N_{tr}int(M \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Hence M is $N_{tr}\Upsilon$-open.

Theorem 3.41. *If L is a $N_{tr}\Upsilon$-open set, then $N_{tr}cl(L)$ is N_{tr}semi-open.*
Proof. By theorem 3.23, L is $N_{tr}\beta$-open. Then $L \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$ which implies $N_{tr}cl(L) \subseteq N_{tr}cl(N_{tr}cl(N_{tr}int(N_{tr}cl(L)))) = N_{tr}cl(N_{tr}int(N_{tr}cl(L)))$. Hence $N_{tr}cl(L)$ is N_{tr}semi-open.

Theorem 3.42. *If L is a N_{tr}pre-open set, then $N_{tr}cl(L)$ is $N_{tr}\Upsilon$-open.*
Proof. Since L is N_{tr}pre-open, we have $L \subseteq N_{tr}int(N_{tr}cl(L))$ which implies $N_{tr}cl(L) \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L))) \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L \cup F))) \subseteq N_{tr}cl(N_{tr}int(N_{tr}cl(L) \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Therefore $N_{tr}cl(L)$ is $N_{tr}\Upsilon$-open.

4. Neutrosophic Υ-closed Sets

Analogous to the previous section, this section deals with neutrosophic Υ-closed sets and its features are presented concisely.

Definition 4.1. *A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be neutrosophic Υ-closed if for every non-empty N_{tr}open set $G \neq 1_{N_{tr}}$, $N_{tr}int(N_{tr}cl(L \cap G)) \subseteq L$. In other words, the complement of a neutrosophic Υ-open set is neutrosophic Υ-closed. The class of neutrosophic Υ-closed sets is denoted by $N_{tr}\Upsilon C(U, \tau_{N_{tr}})$.*

Example 4.2. Consider the neutrosophic topological space $(U, \tau_{N_{tr}})$ in example 3.2. Let $\mathcal{Q} = \{P^c \colon P \in \mathcal{P}\}$. Then $N_{tr}\Upsilon C(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L^c, M^c, \mathcal{Q}, 1_{N_{tr}}\}$.

Theorem 4.3. *In any neutrosophic topological space $(U, \tau_{N_{tr}})$,*

i. *Every N_{tr}closed set is $N_{tr}\Upsilon$-closed.*

ii. *Every N_{tr}semi-closed set is $N_{tr}\Upsilon$-closed.*

iii. *Every $N_{tr}\alpha$-closed set is $N_{tr}\Upsilon$-closed.*

iv. *Every $N_{tr}\Upsilon$-closed set is $N_{tr}\beta$-closed.*

v. *Every $N_{tr}\Upsilon$-closed set is $N_{tr}gs$-closed.*

vi. *Every $N_{tr}\Upsilon$-closed set is $N_{tr}gb$-closed.*

Proof. Proof is obvious.

Remark 4.4. *The converse of each of the statements of the above theorem need not be true.*

Remark 4.5.

i. *The intersection of an arbitrary collection of $N_{tr}\Upsilon$-closed sets is $N_{tr}\Upsilon$-closed.*

ii. *The union of any two $N_{tr}\Upsilon$-closed sets need not be $N_{tr}\Upsilon$-closed.*

iii. *The concepts of $N_{tr}\Upsilon$-closed and N_{tr} pre-closed are independent.*

iv. *The concepts of $N_{tr}\Upsilon$-closed and $N_{tr}g$-closed are independent.*

v. *The concepts of $N_{tr}\Upsilon$-closed and $N_{tr}b$-closed are independent.*

vi. *The concepts of $N_{tr}\Upsilon$-closed and $N_{tr}\psi$-closed are independent.*

vii. *The class of $N_{tr}\Upsilon$-closed sets lie between N_{tr}semi-closed sets and $N_{tr}gs$-closed sets.*

5. Neutrosophic Υ-interior and Υ-closure

In this section, we define and examine the attributes of neutrosophic Υ-interior and closure operator.

Definition 5.1. *Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space and L be a neutrosophic set in U.*

i. *The neutrosophic Υ-interior of L is the union of all $N_{tr}\Upsilon$-open sets contained in L. It is denoted by $N_{tr}\Upsilon int(L)$.*

ii. *The neutrosophic Υ-closure of L is the intersection of all $N_{tr}\Upsilon$-closed sets containing L. It is denoted by $N_{tr}\Upsilon cl(L)$.*

Remark 5.2.

i. *$N_{tr}\Upsilon int(L)$ is the largest $N_{tr}\Upsilon$-open set contained in L.*

ii. *$N_{tr}\Upsilon cl(L)$ is the smallest $N_{tr}\Upsilon$-closed set containing L.*

Theorem 5.3. *Let L be a neutrosophic set of a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then,*

i. *L is $N_{tr}\Upsilon$-open if and only if $N_{tr}\Upsilon int(L) = L$.*

ii. *L is $N_{tr}\Upsilon$-closed if and only if $N_{tr}\Upsilon cl(L) = L$.*

Proof. (i) Suppose L is $N_{tr}\Upsilon$-open. Then, by definition 5.1(i) , it is obvious that $N_{tr}\Upsilon int(L) = L$. Conversely, suppose $N_{tr}\Upsilon int(L) = L$. Then, by remark 5.2(i) , $N_{tr}\Upsilon int(L)$ is $N_{tr}\Upsilon$-open and hence L is $N_{tr}\Upsilon$-open.
(ii) The proof is similar to (i).

Theorem 5.4. *Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space and L, M be neutrosophic sets in U. Then*

i. *$N_{tr}\Upsilon int(0_{N_{tr}}) = 0_{N_{tr}}$ and $N_{tr}\Upsilon int(1_{N_{tr}}) = 1_{N_{tr}}$*

ii. *$L \subseteq M \Longrightarrow N_{tr}\Upsilon int(L) \subseteq N_{tr}\Upsilon int(M)$*

iii. *$N_{tr}\Upsilon int(N_{tr}\Upsilon int(L)) = N_{tr}\Upsilon int(L)$*

iv. *$N_{tr}\Upsilon int(L \cup M) \supseteq N_{tr}\Upsilon int(L) \cup N_{tr}\Upsilon int(M)$*

v. *$N_{tr}\Upsilon int(L \cap M) \subseteq N_{tr}\Upsilon int(L) \cap N_{tr}\Upsilon int(M)$*

Proof. (i) $0_{N_{tr}}$ and $1_{N_{tr}}$ are $N_{tr}\Upsilon$-open sets. Hence by theorem 5.3, $N_{tr}\Upsilon int(0_{N_{tr}}) = 0_{N_{tr}}$ and $N_{tr}\Upsilon int(1_{N_{tr}}) = 1_{N_{tr}}$
(ii) By remark 5.2(i), $N_{tr}\Upsilon int(L) \subseteq L$ and $N_{tr}\Upsilon int(M) \subseteq M$. Now, $N_{tr}\Upsilon int(L) \subseteq L \subseteq M$ implies $N_{tr}\Upsilon int(L) \subseteq M$. Since $N_{tr}\Upsilon int(M)$ is the largest $N_{tr}\Upsilon$-open set contained in M, we have $N_{tr}\Upsilon int(L) \subseteq N_{tr}\Upsilon int(M)$.
(iii) By remark 5.2(i), $N_{tr}\Upsilon int(L)$ is $N_{tr}\Upsilon$-open. Then by theorem 5.3, $N_{tr}\Upsilon int(N_{tr}\Upsilon int(L)) = N_{tr}\Upsilon int(L)$.
(iv) Since $L \subseteq L \cup M$, it follows from (iii) that $N_{tr}\Upsilon int(L) \subseteq N_{tr}\Upsilon int(L \cup M)$. Similarly, $N_{tr}\Upsilon int(M) \subseteq N_{tr}\Upsilon int(L \cup M)$. Hence $N_{tr}\Upsilon int(L \cup M) \supseteq N_{tr}\Upsilon int(L) \cup$

$N_{tr}\Upsilon int(M)$.
(v) Since $L \cap M \subseteq L$, it follows from (iii) that $N_{tr}\Upsilon int(L \cap M) \subseteq N_{tr}\Upsilon int(L)$. Similarly, $N_{tr}\Upsilon int(L \cap M) \subseteq N_{tr}\Upsilon int(M)$. Hence, $N_{tr}\Upsilon int(L \cap M) \subseteq N_{tr}\Upsilon int(L) \cap N_{tr}\Upsilon int(M)$.

Remark 5.5. *The above theorem is true for neutrosophic Υ-closure.*

Theorem 5.6. *For any neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$,*

i. $(N_{tr}\Upsilon int(L))^c = N_{tr}\Upsilon cl(L^c)$

ii. $(N_{tr}\Upsilon cl(L))^c = N_{tr}\Upsilon int(L^c)$

Proof. (i) $(N_{tr}\Upsilon int(L))^c = (\cup\{M : M \subseteq L \text{ and } M \in N_{tr}\Upsilon O(U, \tau_{N_{tr}})\})^c$
$= \cap\{M^c : L^c \subseteq M^c \text{ and } M^c \in N_{tr}\Upsilon C(U, \tau_{N_{tr}})\}$
$= N_{tr}\Upsilon cl(L^c)$
(ii) Proof is similar to (i).

Theorem 5.7. *Let L be a neutrosophic set of a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then,*

i. $N_{tr}\Upsilon int(L) \subseteq L \cap N_{tr}cl(N_{tr}int(L \cup F))$ *for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$.*

ii. $N_{tr}\Upsilon cl(L) \supseteq L \cup N_{tr}int(N_{tr}cl(L \cap G))$ *for every non-empty N_{tr}open set $G \neq 1_{N_{tr}}$.*

Proof. (i) By remark 5.2(i), $N_{tr}\Upsilon int(L) \subseteq N_{tr}cl(N_{tr}int(N_{tr}\Upsilon int(L) \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$. Now, since $N_{tr}\Upsilon int(L) \subseteq L$, $N_{tr}\Upsilon int(L) \subseteq N_{tr}cl(N_{tr}int(L \cup F))$ and hence $N_{tr}\Upsilon int(L) \subseteq L \cap N_{tr}cl(N_{tr}int(L \cup F))$ for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}$.
(ii) By remark 5.2(ii), $N_{tr}\Upsilon cl(L) \supseteq N_{tr}int(N_{tr}cl(N_{tr}\Upsilon cl(L) \cap G))$ for every non-empty N_{tr}open set $G \neq 1_{N_{tr}}$. Now, since $L \subseteq N_{tr}\Upsilon cl(L)$, $N_{tr}\Upsilon cl(L) \supseteq N_{tr}int(N_{tr}cl(L \cap G))$ and hence $N_{tr}\Upsilon cl(L) \supseteq L \cup N_{tr}int(N_{tr}cl(L \cap G))$ for every non-empty N_{tr}open set $G \neq 1_{N_{tr}}$.

6. Conclusion

The neutrosophic theory has entrenched as a significant mathematical tool in dealing with uncertainties. The liberal attribute of the membership functions in neutrosophic sets has made it more flexible than fuzzy and intuitionistic fuzzy sets. This work instigated a new class of sets in neutrosophic topological spaces and discussed its attributes. Further, it has been compared with other existing sets and the observations are delineated through results and illustrations. The interior and closure operators are also defined by means of the new class of sets. This idea can be exerted in the study of various topological concepts such as continuity, compactness, connectedness and separation axioms in the neutrosophic environment.

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